Looking for Linearity: Integrating Graphing for First-Year Chemistry Students

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Abstract: Based on our observation that the general literature does not provide an organizing principle for the graphs that science students encounter, an approach called "Looking for Linearity" has been described. This approach is based on the hypothesis that when scientists look at their data and begin to represent it, they initially look for linearity. This is to say that scientists use Occam's Razor; variables are used and transformed in such ways that when plotted against each other, the simplest representation—the straight line—is produced. A brief review of the topics typically covered in the first year of chemistry reveal a substantial number of relationships either expressed in the form of a straight line (gas laws, free energy, rate laws) or in terms of ratios that when graphed produce straight lines (density, specific heat capacity, stoichiometry). "Looking for Linearity" is an approach to graphing that serves four purposes for teaching first year chemistry students: 1) it weaves a common theme or thread through the entire year of General Chemistry, 2) it allows students to work like scientists, 3) it connects an important mathematical construct with chemical concepts, and 4) it provides a method to process data in other scientific fields like physics. The linearity heuristic is represented in what is called a Graphing Decision Tree. This tree shows, in simplified terms, how linearity can be used to organize different types of graphs found in the first year of chemistry. The Decision Tree is hierarchically structured from simple to increasing graphing complexity. Straight lines were listed as being the simplest to interpret, followed by exponential curves and then non-exponential curves; exponential curves were second because they could be converted to straight lines by using logarithms. Each pathway ends with examples of some of the different types of graphs our students will encounter in the first year of chemistry.

Every chemist knows that graphing is an invaluable method of representing data and determining relationships between variables. In the first-year general chemistry course, students construct many different types of graphs, or come upon them as they read the chapters in their textbook. It is our observation that neither college chemistry texts nor the general literature provide an organizing principle for the graphs that our science and premedical majors encounter. For example, there is no discussion on integrating the different types of graphs applicable to introductory chemistry, there is no mechanism by which students can rank the importance of these graphs, nor is there an explicit account of graphing as having a unifying purpose. While a review of the mathematical education literature clearly underscores the importance of graphing, much of the educational research centers not on college students, but on elementary and secondary students and involves their concept formation [2–6], achievements with microcomputer-based labs and calculators [7–12], and their interactions with instructional strategies [13–16]. Little is said about the organization of graphs in college science, and nothing is said about how graphs in a general chemistry course can be incorporated into a meaningful whole.

Spurred on by this observation, we started to look at the process of graphing data from the perspective of the scientist. Our hypothesis is that when scientists look at and begin to represent their data, they initially look for linearity. This is to say that scientists use Occam's Razor; variables are used and transformed in such ways that, when plotted against each other, the *simplest* representation—the straight line—is produced. We believe that this tacit concern or impetus to linearize data is one of the processes that scientists rely on to solve their problems. A brief review of the topics typically covered in the first year of chemistry reveals a substantial number of relationships either expressed in the form of a straight line (gas laws, free energy, rate laws) or in terms of ratios that when graphed produce straight lines (density, specific heat capacity, stoichiometry).

Looking for Linearity is an approach to graphing that serves four purposes in teaching the first-year chemistry student: 1) it weaves a common theme or thread through the entire year of a general chemistry course, 2) it allows students to work like scientists, 3) it connects an important mathematical construct with chemical concepts, and 4) it provides a method to process data in other scientific fields, such as physics. The advantages of using linearization in a laboratory context have been briefly discussed [17], as well as the problems as they relate to computer graphing programs [18].

The linearity heuristic that we have developed is an important piece of our graphing curriculum. An abridged sequence of topics on this subject can be seen below.

- Variables and ratios
- Graphing nomenclature

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- Plotting
- Linear graphs, directly proportional with *y*-intercept equal to zero
- Finding the slope; finding constants (e.g., density)
- Linear graphs, inversely proportional
- Linear graphs with *y*-intercept equal to a nonzero number
- Finding the slope; finding universal constants (e.g., *R,* the gas law constant)
- Graphs that deviate from linearity at specific conditions
- Exponential graphs
- Drawing tangents to an exponential curve
- Identifying equations that can be expressed in the form $y = mx + b$

A Graphing Decision Tree

The linearity concept is represented in what we call a *Graphing Decision Tree*. This tree shows, in simplified terms, how linearity can be used to organize different types of graphs found in the first year of chemistry (Fig. 1). It is hierarchically structured from simple to increasing graphing complexity. Straight lines are the simplest to interpret, followed by exponential curves, and then nonexponential curves; exponential curves are second because they can be converted to straight lines by using logarithms. Each pathway ends with subject headings that relate to examples of some of the different types of graphs our students will encounter in the first year of chemistry. This tree, as it is represented, is just a part of our graphing curriculum; additional information such as equations, graphs with nonzero intercepts, and other examples could be included if necessary. While it might not be appropriate for most first-year chemistry students, readers should be aware that a preferred approach to data analysis is nonlinear regression, as opposed to using transformations such as logarithms to create straight lines. Nonlinear regression programs give more realistic weight to the x–y data in the calculation of uncertainties that are applied to such values as *A, B,* and *C* found in the theoretical equation, $y = A + Bx + Cx^2$.

Using the Tree in the Classroom

In our interdisciplinary physics/chemistry/math course, the concept of linearity is introduced in the context of a lecture/laboratory module on gases [19]. In this module, students use spreadsheets to organize and represent their data, and then apply the linearity rule in their first attempt to discover a relationship between variables. Once a model has been developed for the behavior of gases, students use the mathematics inherent in the model to search for linear patterns when new concepts and variables are introduced. Students are taught a variety of different ways to linearize data, from taking an inverse to using logarithms. Once these strategies are mastered, students are given a handout similar to the one in Figure 1, with the exception that the lower tier boxes are empty. Students fill in these boxes with examples that they have previously encountered, along with new graphing examples, such as when they study Beer's law in spectroscopy, titration curves in acid-base equilibria, Gibbs free energy in thermodynamics, and the Nernst equation in electrochemistry.

Data to be graphed can be generated from laboratory experiments, or be given to students as a problem-solving activity to do in class or for homework. For example, in a typical density experiment, students can readily plot their mass data versus the volume of a substance, and then draw upon the formula for a straight line to determine the slope, or density of the substance they are studying. In the more sophisticated problems, teachers can give the form of the variables to be plotted. For example, students are unlikely to discover that the plot of ln *k* vs 1/T associated with the Arrhenius equation would produce a straight line. Therefore, it is appropriate for teachers to supply students with instructions detailing the specific form of the variables that must be plotted. In other cases, an equation is given and students asked to find experimentally a constant such as E_a in the kinetics equation:

$$\ln(k) = \frac{-E_a}{R}\left(\frac{1}{T}\right) + \ln(A)$$

E_a, which represents activation energy, can be determined by finding the slope of the line plotted from an appropriate relationship between rate constants (*k)* and temperature (*T)*; the *y*-intercept is the natural logarithm of the so-called frequency factor. By following this approach, we believe that students will come to understand that data and their graphical representations are central to science, that linearity is a powerful tool that scientists use to understand their data, and, lastly, that graphing can allow other parameters to be determined when the relationships between variables are known.

While we anticipate conducting research to determine the extent to which students use this heuristic, and how it has affected their attitudes towards graphing, some anecdotal evidence has been collected. For example, some introductory chemistry students have remarked that the organization of so many graphs into one readable format alleviated the anxiety or fear when faced with graph construction and interpretation. Some students also pointed out that the tree should "not be followed blindly"; the reasons why one is graphing should not be forgotten. One student recommended that the class write an essay on $y = mx + b$. When this heuristic was introduced to in-service secondary science teachers during a graduate-level methods course, some other concerns were raised. One teacher thought that a computer program would be a better medium to convey the tree, with links to visual examples of specific graphs. Another teacher recommended the construction of a tree devoted to the interpretation of graphs. In terms of benefits, two common ideas were voiced: 1) teachers liked the fact that a great number of graphs were presented and that the pathways were clear to follow, and 2) many teachers believed that this map would be best used as a way to summarize their students' knowledge of the different types of graphs they encountered.

Conclusion

In closing this article, we would like to qualify our purpose. We are not saying that students should force linearity onto their plots regardless of the shape that their plots suggest. Rather, we contend that linearity has some important advantages, and should be looked for and thought about at the onset of graphing activities. Furthermore, linearity is one way in which different graphs that science students come upon can

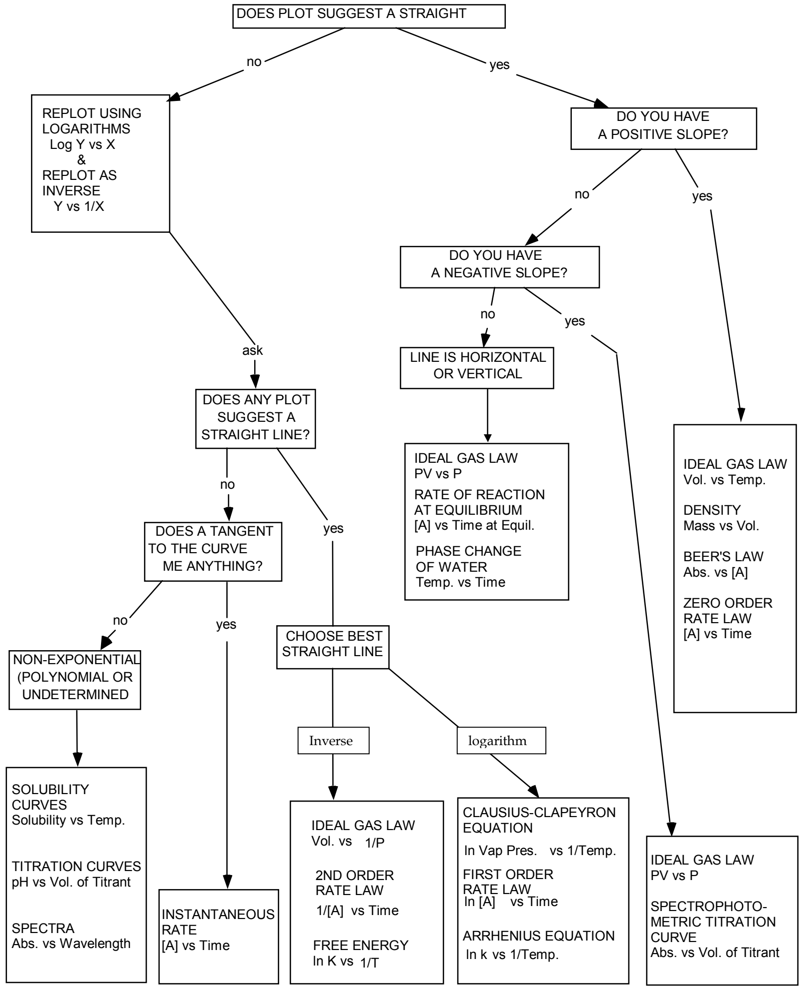

Figure 1. Graphing Decision Tree

be organized; it is not the only way. We clearly emphasize to our students that the advantage of straight lines lies with the relative ease of determining a mathematical relationship between two variables, in comparison to irregular curves such as those generated from acid–base titrations. We anticipate that given this process of thinking about graphing, students will act like scientists, be better equipped to make those difficult transitions between data, graphical representations, and mathematical equations, and finally be able to integrate their graphing skills through the year-long general chemistry course. While looking for linearity might seem obvious to readers of this and other chemical education journals, it is not obvious to first-year chemistry students, some of whom will ultimately become our future's scientists. In short, the *Graphing Decision Tree* makes the implicit explicit, and by so doing, ties together discrete information into a thematic whole.

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